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A Framework for Economic Growth with Capital-Embodied Technical Change†

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Technological advance is often embodied in capital inputs, like computers, airplanes, and robots. This paper builds a framework where capital inputs advance through (i) increased automation and (ii) increased productivity. The interplay of these two innovation dimensions can produce balanced growth, satisfying the Uzawa Growth Theorem even though technological progress is capital-embodied. The framework can further address structural transformation, general-purpose technologies, the limited macroeconomic impact of computing, and declining productivity growth and labor shares. Overall, this tractable framework can help resolve puzzling tensions between micro-level observations of innovation and balanced growth while providing new perspectives on numerous macroeconomic phenomena. (*JEL* E22, E23, E24, E25, L16, O33, O41)

This paper provides a conceptual framework to address key tensions between microeconomic descriptions of technological progress and the macroeconomic features of economic growth. Specifically, the model connects three features that usually have trouble sitting together: (i) capital-embodied technical change, (ii) balanced growth, and (iii) a nonunitary elasticity of substitution between capital and labor. The model provides a tractable and intuitive approach for thinking about growth rates, income shares, industry dynamics, and structural change, all based in forms of capital-embodied technical change. At the heart of the model is a surprise: a framework that can feature purely capital-embodied innovations at a micro level yet, at a macro level, purely labor-augmenting technological change.

To motivate this paper, consider first that major forms of technological progress appear to be embodied in physical capital. For example, advances in transportation have followed innovations in capital equipment (e.g., engines; also trains, automobiles, and airplanes). Advances in agriculture appear to follow capital-embodied innovations in machines (e.g., tractors and combine harvesters) and other nonlabor inputs (e.g., seeds, fertilizers, and pesticides). Advances in manufacturing also appear in

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many capital-embodied forms (e.g., robots, photolithographic machines, 3-D printers). Mokyr's classic *Lever of Riches* more generally studies productivity-enhancing innovations through human history, and it would be hard from this telling not to conclude that capital-embodied innovations are at the heart of technological progress (Mokyr 1990). Indeed, the major "general-purpose technologies" (Bresnahan and Trajtenberg 1995; Helpman 1998) since the Industrial Revolution—engines, electrification, computing, and now perhaps artificial intelligence—are all embodied in capital equipment. See Figure 1.

While the importance of capital-embodied technical change seems self-evident, it creates surprising tensions for models of economic growth. To match the key stylized facts of a balanced growth path (Kaldor 1961), the Uzawa Growth Theorem shows that technological progress in the aggregate production function must be purely labor augmenting (Uzawa 1961). Namely, write aggregate production as

$$Y_t = F(B_t K_t, A_t L_t), \tag{1}$$

where Y_t, K_t, and L_t are aggregate output, capital, and labor, respectively. The Uzawa Growth Theorem requires the capital-augmenting technology term to be constant, $B_t = B$, with per capita income growth following only from an increasing labor-augmenting technology term, A_t. This result is surprising, even paradoxical, given that technological advance at a micro level appears substantially embodied in capital, and yet a balanced growth path seems a reasonable description of the economy as a whole.

The traditional "fix" in growth models has been to utilize a Cobb-Douglas aggregate production function, a special case where capital-augmenting technological advance can be recast as labor augmenting and balanced growth is indifferent to the source of technological advance. However, this special case raises additional tensions. Namely, there is substantial evidence for a nonunitary elasticity of substitution between capital and labor, and especially for an elasticity that is less than one (e.g., Antràs 2004; Chirinko 2008; Oberfield and Raval 2021; Grossman et al. 2021). Further, a Cobb-Douglas approach locks the labor share of income to an exogenous parameter, limiting its capacity to inform pertinent contemporary issues, including the apparent decline in the labor share (e.g., Elsby, Hobijn, and Sahin 2013; Karabarbounis and Neiman 2014; Dao et al. 2017).

This paper works to resolve these tensions, developing a conceptual framework where technological advance can be embodied in capital inputs at a micro level yet match the macroeconomic regularities. The key idea is that there are two types of capital-embodied technological advance. Both margins operate in a task-based model of the economy. The first is an extensive margin of advance: automating activities or tasks that were formerly performed by labor (Zeira 1998; Autor, Levy, and Murnane 2003; Acemoglu and Autor 2011; Acemoglu and Restrepo 2018, 2020). The second is an intensive margin of advance: improving the productivity of capital at the already-automated tasks. Ultimately, the economy depends on two technology state variables. The first, β_t, is the share of tasks in the economy that are automated (i.e., performed by capital inputs). The second, Z_t, is an index of productivity across these capital inputs. So, for example, a new application of computers, replacing labor at a task, causes β_t to go up, while faster computers cause Z_t to go

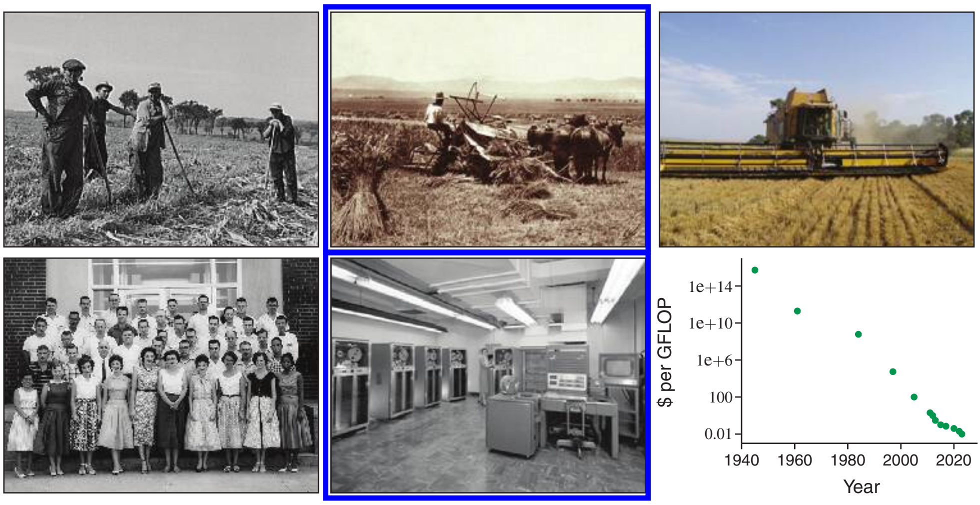

Figure 1

Notes: Top: Agricultural workers working with limited tools (left). A mechanical reaper and binder from the 1890s (middle). A modern combine harvester (right). A single Lexion 760 Terra Trac combine harvester in 2018 harvested 2.6 million kilograms of corn in 12 hours. Bottom: "Computers" at NASA in the 1950s (left). The IBM 704 mainframe computer, introduced in 1954 (middle). The declining cost of computer operations per second, falling by 10^{16} over the last 75 years (right).

Source: All images are in the public domain, with the combine harvester image credited to Ada Macey with creative commons license cc-by-2.0, and the IBM 704 image credited to Lawrence Livermore National Laboratory. Computing costs are plotted by the authors using data from https://en.wikipedia.org/wiki/FLOPS.

up. Ultimately, the capital share proves to be a remarkably simple outcome based on these two technology indices:

$$\text{(2)} \qquad \textit{Capital Share} \;=\; \beta_t Z_t^{-1},$$

which situates the key intuition. Automation causes the capital share to rise (Acemoglu and Restrepo 2018, 2020). However, advances in the productivity of the capital inputs cause the capital share to fall (Aghion et al. 2019). This latter effect follows from the final key to the model: that the elasticity of substitution between capital and labor is less than one.[1] In this context, advances in productivity of a given input cause its price to fall relatively quickly, so that its share in GDP declines. Thus, the two technology indices end up pushing in opposite directions with regard to the capital share and can lead to balanced growth.[2]

The approach builds on recent advances in the growth literature. The first advances are models of automation, which have extended how we conceptualize technological progress (e.g., Zeira 1998; Acemoglu and Restrepo 2018). In contrast to the classic vertical approaches where innovations are modeled as directly

[1] The elasticity of substitution between capital and labor is debated (e.g., Chirinko 2008; Karabarbounis and Neimann 2014), but the balance of the empirical literature appears to favor an elasticity of substitution less than one (e.g., reviews by Hamermesh 1993 and Chirinko 2008; Antràs 2004; Oberfield and Raval 2021, among others). See also Grossman et al. (2021).

[2] The possibility that a balanced growth path might emerge along these lines was first suggested, to our knowledge, in Aghion, Jones, and Jones (2019).

productivity enhancing (i.e., a technology term weighting the quantity of the input or the cost of producing the input), automation allows us to consider a horizontal form of innovation where one input (capital) replaces another (labor). It is the combination of the classic view and the automation view that makes the model in this paper work. A second new strand of literature, which focuses more precisely on the Uzawa Theorem, introduces human capital into the production function in a way that holds the capital-augmenting term constant along the balanced growth path (Grossman et al. 2017, 2021). In the Grossman et al. (2017, 2021) approach, there is capital-augmenting technological progress, but human capital in a sense eats this physical capital with the result that the capital-augmenting term B_t can remain constant as both capital technology advances and human capital deepens.

This paper relates to both ideas but with distinct foundations, forces, and intuitions, and a wide set of applications and results. First, at the aggregate level, the two forms of capital advancement act as neutralizing forces in the aggregate production function. In particular, when aggregating from the task level up to total output, the technology indexes enter the aggregate production function in the form

$$Y_t = F\Big(\big(\beta_t Z_t^{-1}\big)^{\theta} K_t, (1-\beta_t)^{\theta} L_t\Big), \tag{3}$$

where $\theta < 0$, which further demonstrates key intuition. Essentially, the extensive and intensive margins of technical progress are in a "tug-of-war" with each other in the aggregate. Balanced growth emerges when advances in automation (e.g., computers take over more tasks) and advances in the productivity of capital inputs (e.g., computers get faster) proceed at the same rate. This holds the capital share constant in (2), and, as we see in (3), meets the key Uzawa condition that the capital-augmenting term be constant in the production function to achieve balanced growth. Further, while these two capital-technology indices neutralize each other in the overall capital technology index, the balanced growth path is still driven by the capital-embodied improvements, as labor focuses on and increases production in the remaining nonautomated tasks and capital deepens overall.

Second, in addition to being able to meet the requirements of the Uzawa Growth Theorem, this tractable framework provides novel and intuitive perspectives for how technology dynamics and economic dynamics can relate. For example, a decline in the labor share follows naturally if automation accelerates or vertical improvements slow. Further, a slowdown in vertical technological progress leads both to a labor share decline and a growth slowdown, providing a potentially straightforward conceptual linkage between the observation that innovation may be getting harder (e.g., Jones 1995; Jones 2009; Bloom et al. 2020) and recent US growth and income share observations.

Third, viewing different intervals of the economy's tasks as representing different sectors, extensive and intensive advances in capital equipment (and the tug-of-war between them) can occur differently within different sectors, allowing for distinct industry dynamics and structural change along a steady growth path. This process occurs in line with Baumol's cost disease, where high-productivity sectors tend to shrink as a share of GDP and laggard sectors grow as a share of GDP (Baumol 1967; Aghion et al. 2019). For example, if agriculture and manufacturing see relatively rapid technological advance compared to services, then the GDP share of agricultural outputs and manufacturing outputs will go down and the GDP share of services will go up. Yet the capital share in agriculture or manufacturing need not go up or down compared to services, and the capital share in the economy can remain steady. Thus, this model can speak to structural change as well as balanced growth.

As another application, take computers. Moore's Law, which has increased floating-point operations per second by 10^{11} since World War II, is often seen as an essential technological force of our age, and yet overall productivity growth has been modest, and perhaps puzzlingly so (Solow 1987). A Baumol perspective can naturally help: capital-embodied tasks (e.g., floating-point operations) that advance rapidly become a smaller share of GDP, and growth becomes more determined by productivity at other, bottleneck tasks. Thus, the very success of Moore's Law engenders its ultimate impotence in the aggregate production function, if it only applies to a limited set of tasks (see also Aghion et al. 2019). However, at the same time, and unlike agriculture, computing equipment has not become a smaller share of GDP. Rather, ICT capital equipment investment tends to sustain at high levels in advanced economies (OECD 2021). The model suggests that this sustained GDP share occurs through increasing automation: if computers are simultaneously taking over more tasks (e.g., search, machine control, artificial intelligence, etc.). This increasing breadth of tasks undertaken by computers can balance computer productivity advances at given tasks and sustain computers' GDP share. The model may then also address both the power and the limits of computing, among other general-purpose technologies.

This paper proceeds as follows. Section I develops the baseline model with exogenous technological advances on both the extensive and intensive margins and considers conditions for a balanced growth path. Section II uses this model to examine broader technology dynamics and economic outcomes, with applications to dynamics in the overall growth rate and income shares, structural change, and general-purpose technologies. Section III then develops an endogenous growth model, where R&D investment and rates of progress on both technology margins are choices by firms and a balanced growth path emerges based on fundamental economic parameters. Additional applications are then discussed, including further interpretations for the apparent productivity slowdown and declining labor share of income. Section IV concludes.

I. Baseline Growth Model

Here, we present the growth model with exogenous technological progress. We first lay out the assumptions regarding production and preferences and consider equilibrium based on consumer and firm optimization. We then consider a balanced growth path (BGP). Namely, we will show the existence of a BGP—even though all technological progress is embodied in capital goods and the elasticity of substitution is less than one.

A. *Production Technology*

The production technology features a unit interval of tasks, $i \in [0,1]$. Final output Y_t is given as

$$Y_t = \upsilon\left[\int_0^1 y_t(i)^\rho \, di\right]^{1/\rho}, \quad \rho < 0, \tag{4}$$

where $y_t(i)$ is an intermediate good. By assumption, $\rho < 0$, so that intermediates are gross complements. The price of final output is the numeraire.

Intermediate good production is as follows. Each intermediate good can be produced with labor. However, intermediate goods on the interval $i \in [0, \beta_t]$ can also be produced by capital. The production possibilities for the intermediates are

$$y_t(i) = \begin{cases} A l_t(i), & \text{for all } i \in [0,1]; \\ z_t(i)^{\frac{\rho-1}{\rho}} x_t(i), & \text{for all } i \in [0,\beta_t]; \end{cases} \tag{5}$$

where $l_t(i)$ is a labor input, $x_t(i)$ is a capital input, and $z_t(i)$ is a capital-input specific productivity term. The capital input can be made at a marginal cost of ψ units of the final good. Following standard vertical growth models (e.g., Aghion and Howitt 1992), we assume capital inputs depreciate fully with their use, and we will normalize the cost of machine production such that $\psi = \upsilon$. The exponent on the capital-productivity term is for notational convenience (and is positive since $\rho < 0$).[3]

Capital-embodied technology in this model is thus described on two key margins. First, there is the share of tasks, β_t, that have been automated. Second, there is the productivity, $z_t(i)$, of each automated task. Note as well that we are fixing the labor productivity via the constant A. This feature can easily be relaxed, but we fix labor productivity here to emphasize that all technological progress in this model can be embodied in capital—and yet we will have a balanced growth path. Figure 2 depicts the two capital-embodied technology frontiers, in productivity and automation, in this model.

Finally, it will be useful to summarize the capital-productivity terms using the index

$$Z_t = \left[\frac{1}{\beta_t}\int_0^{\beta_t} \frac{1}{z_t(i)} di\right]^{-1}, \tag{6}$$

which is the harmonic average of the capital-embodied technology terms.

B. *Preferences*

A representative household has CRRA preferences,

$$U(t) = \int_t^{\infty} \frac{c(\tau)^{1-\theta} - 1}{1-\theta} e^{-\omega(\tau - t)} d\tau, \tag{7}$$

where ω is the discount rate and $\theta \geq 0$. A household supplies one unit of labor inelastically to production, earning a wage rate w_t. Households earn wages as well as earn any capital income, holding a balanced portfolio of the firms in the economy. We assume the usual transversality condition. Consumption will proceed according to the Euler equation

$$g_c = \frac{1}{\theta}(r_t - \omega). \tag{8}$$

[3] One could alternatively write automated intermediates production as $y_t(i) = \hat{z}_t(i) x_t(i)$, and readers interested in that alternative can make the substitution $z_t = \hat{z}_t(i)^{\rho/(\rho-1)}$ in what follows.

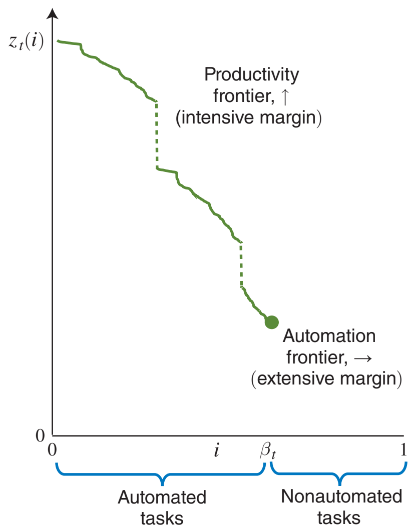

Figure 2. The Two Technology Frontiers in the Model

C. *Resource Constraints*

The total supply of labor is L_t, where

$$L_t = \int_0^1 l_t(i)di, \tag{9}$$

and L_t grows at rate n. The total investment cost for capital is

$$I_t = \int_0^1 \psi x_t(i)di. \tag{10}$$

Output is either consumed or used to make capital inputs, giving the economy-wide constraint

$$I_t + C_t = Y_t,$$

where $C_t = L_t c_t$ is total consumption.

D. *Firm Optimization*

Competitive final goods firms maximize profits using the final goods production function (4) and taking prices as given. These producers' demand for intermediate goods will thus take the form

$$p_t(i) = \upsilon^\rho \left[\frac{Y_t}{y_t(i)}\right]^{1-\rho}. \tag{11}$$

Intermediate firms maximize profits using the production possibilities (5), taking intermediate output prices, $p_t(i)$, and input prices for labor and capital as given.

Nonautomated Tasks.—Given the intermediate production technology (5) and competitive markets, the intermediate price for any nonautomated task is

$$p_t(i) = w_t/A. \tag{12}$$

Intermediate prices are thus the same across all labor-performed tasks. From (11), it then follows that $y_t(i)$ is the same for these labor-performed tasks. We will be interested in the equilibrium where automated tasks (on the interval $i \in [0, \beta_t]$) will indeed use capital inputs, so that labor input is used only on tasks $i \in (\beta_t, 1]$. Anticipating this feature, and using the labor resource constraint (9), we then have the equilibrium labor allocation $l_t(i) = L_t/(1 - \beta_t)$ and outputs $y_t(i) = (AL_t)/(1 - \beta_t)$ for labor-produced tasks.

Automated Tasks.—For tasks $i \in [0, \beta_t]$, firms can use capital inputs or labor inputs. For the exogenous growth model, these intermediate producers will be competitive and earn zero profits. If firms use capital inputs, the prices are

$$p_t(i) = \psi z_t(i)^{\frac{1-\rho}{\rho}}, \quad i \in [0, \beta_t]. \tag{13}$$

To focus on the equilibrium of interest, where these firms do indeed use capital inputs, we require as a technical condition that automated production is lower cost than labor-based production for automated tasks; i.e.,

$$w_t \geq \psi A z_t(i)^{\frac{1-\rho}{\rho}} \tag{14}$$

for all automated technologies $i \in [0, \beta_t]$. We will validate this technical condition later.

For automated tasks, the equilibrium capital allocation to each task is, using (11), (13), and the production technology (5),

$$x_t(i) = \upsilon^{-1} \frac{Y_t}{z_t(i)}, \quad i \in [0, \beta_t]; \tag{15}$$

the intermediate outputs are

$$y_t(i) = \upsilon^{-1} z_t(i)^{-\frac{1}{\rho}} Y_t, \quad i \in [0, \beta_t]; \tag{16}$$

and the GDP share for a given automated task is $p_t(i) y_t(i)/Y_t = 1/z_t(i)$.

Thus, productivity advances in a given automated task cause its output to go up but its GDP share to go down. This effect is part of Baumol's cost disease, where the GDP share falls for outputs the economy becomes especially good at producing.

E. *Aggregation*

A key feature of the model is how automated intermediates aggregate. Using the capital resource constraint, (10), and the equilibrium capital allocation to each task, (15), we have

$$I_t = Y_t \beta_t Z_t^{-1},$$

where we recall that Z_t is the harmonic average of the tasks-specific productivity terms (see (6)). Recalling that capital inputs fully depreciate in their use, the capital stock K_t is equivalent to total investment each period.

The capital share is then

$$s_{K_t} = \frac{\psi X_t}{Y_t} = \beta_t Z_t^{-1}, \tag{17}$$

and the labor share is

$$s_{L_t} = \frac{w_t L_t}{Y_t} = 1 - \beta_t Z_t^{-1} \tag{18}$$

in terms of the exogenous technology variables. Note that we require $\beta_t Z_t^{-1} < 1$ for well-defined aggregates.

Meanwhile, aggregate output, (4), follows from summing across the equilibrium intermediate outputs. Equilibrium aggregate output is

$$Y_t = \upsilon A \left(1 - \beta_t Z_t^{-1}\right)^{-1/\rho} \left(1 - \beta_t\right)^{\frac{1-\rho}{\rho}} L_t \tag{19}$$

in terms of the exogenous variables.

Equilibrium wages are, from (18) and (19),

$$w_t = \upsilon A \left(1 - \beta_t Z_t^{-1}\right)^{\frac{\rho-1}{\rho}} \left(1 - \beta_t\right)^{\frac{1-\rho}{\rho}}, \tag{20}$$

and consumption per capita is the same as wages, $c_t = w_t$.[4]

F. *Technology Adoption*

Finally, recall that we require automated technologies to be sufficiently productive to be adopted. This condition is (14). Given the equilibrium wage, the technology assumption takes the form

$$z_t(i) \geq z_t^{\min} = \frac{1 - \beta_t}{1 - \beta_t Z_t^{-1}} \tag{21}$$

for all $i \in [0, \beta_t]$, where $z_t^{\min}$ is the minimum productivity level at which an automated task produces the intermediate output at lower cost than labor. Note that,

[4] In this exogenous growth model, there are no profits, and capital depreciates fully in use. Thus, gross investment and capital income are equivalent, and consumption thus tracks wage income.

since the index Z_t is the harmonic average and therefore must be greater than its minimum possible value, $Z_t \geq z_t^{\min}$, it then follows directly from (21) that

$$Z_t \geq 1 \tag{22}$$

along the economy's equilibrium path. We will consider task-level innovation processes that satisfy (21) and create balanced growth further below.

G. *The Balanced Growth Path*

We now focus on a balanced growth path. The BGP is defined as the equilibrium that can match the Kaldor facts. Namely, aggregates Y_t, K_t, and C_t grow at a constant rate, while the interest rate r_t and the capital share of income are constant.

Looking at the aggregate results above, it is apparent that there are two technology conditions for the BGP. First, the BGP requires a constant capital (or labor) share. From (17), we therefore require

$$\beta_t Z_t^{-1} = s_{K_t} = s. \tag{BGP1}$$

Here, we see directly the role of the two forms of technological advance. The capital share is rising in the fraction of sectors that are automated (β_t) and declining in an index of these sectors' productivity (Z_t). These offsetting pressures are exactly why this model can maintain a constant capital share—even though all technological progress is embodied in capital. Note that the capital share–reducing role of capital-embodied productivity gains (Z_t) follows because $\rho < 0$. That is, because intermediate outputs are gross complements, advancing productivity in a sector causes the GDP share of that sector to decline. Thus, the BGP features declining GDP shares of each automated sector. Yet because the share of automated tasks (β_t) is rising, the total share of capital in GDP can remain constant. We will discuss this intuition further below, in light of the Uzawa Theorem.

Second, looking at (19), we see that growth in GDP per capita, g, will also take a simple form. With a constant capital share, $g = \left[(1 - \rho)/\rho\right] g_{1-\beta_t}$. The BGP thus requires a second technology condition. Noting that $(1 - \rho)/\rho$ is negative, steady-state growth occurs when the fraction of nonautomated sectors declines at a constant rate. We write this condition as

$$g_{1-\beta_t} = -q^h, \tag{BGP2}$$

where the h superscript denotes the "horizontal" nature of this innovation, a process of automating tasks. We use a negative sign to emphasize that the share of tasks that are not automated, $1 - \beta_t$, is decreasing as the automation advances. While we will use "horizontal" as an evocative shorthand, note that this mechanism corresponds closely to models of automation and much less to the features and intuition of love-of-variety growth models.[5]

[5] In love-of-variety models, the elasticity of substitution is greater than one, which makes the expansion of variety the central force for aggregate productivity increases. By contrast, the model in this paper features an elasticity

A natural way to think about the condition (BGP2) is that automation is getting harder as it proceeds. The fewer tasks that remain to be automated, the smaller the measures that are successfully automated each period. Anticipating the endogenous growth model, steady-state growth rates will follow from a simple process where innovators seek to automate nonautomated tasks and succeed with probability q^h.

Under the two BGP technology conditions, (BGP1) and (BGP2), the steady-state growth rate of the economy is therefore

$$g = \frac{\rho - 1}{\rho} q^h. \tag{23}$$

Putting the pieces together, on a balanced growth path, the automation rate determines the steady-state growth rate, while the advance of productivity on automated tasks acts to neutralize the effect of automation on the capital share.[6]

We can encapsulate the technology conditions for balanced growth and the balanced growth path as follows.

CONDITION 1: *Let β_t and Z_t grow at the same rate (BGP1). Let $1 - \beta_t$ decline at the rate q^h (BGP2). Further, let the $z_t(i)$ be sufficiently large to satisfy the technology adoption condition (21), and let $\omega - n > (1 - \theta)\big[(\rho - 1)/\rho\big] q^h$ satisfy the transversality condition.*

PROPOSITION 1 (Balanced Growth Path): *Under Condition 1, a balanced growth path exists where the growth rate in per capita output, per capita consumption, the per capita capital stock, and the wage are $g = \big[(\rho - 1)/\rho\big] q^h$. The capital share is $s_{Kt} = s$. The rate of return is $r = \omega + \theta\big[(\rho - 1)/\rho\big] q^h$.*

PROOF:

See Appendix.

Balanced Growth and the Uzawa Theorem.—To gain further insight into the balanced growth path and how the model satisfies the Uzawa steady-state growth theorem, it is helpful to write aggregate output in the form $Y_t = F(B_t K_t, A_t L_t)$. The Uzawa Theorem tells us that, for a balanced growth path with a nonunitary elasticity of substitution between labor and capital, all technological progress must be labor augmenting. That is, we require the labor-augmenting technology term, A_t, to grow at a steady-state rate. Yet the capital-augmenting technology term, B_t, must be constant.

In the model, we can rewrite aggregate output as

$$Y_t = \upsilon \left\{ \left[(\beta_t / Z_t)^{\frac{1-\rho}{\rho}} \upsilon^{-1} K_t \right]^\rho + \left[(1 - \beta_t)^{\frac{1-\rho}{\rho}} A L_t \right]^\rho \right\}^{1/\rho}. \tag{24}$$

of substitution less than one. The measure of tasks is not changing, but the share of tasks done by capital is increasing and the share done by labor decreasing on the balanced growth path.

[6] The technology index Z_t is an aggregation of the $z_t(i)$ (see (6)), and we will consider specific innovation processes for $z_t(i)$ that produce a balanced growth path in Z_t further below.

This formulation shows us directly how the two technology conditions above produce a balanced growth path. First, we see that the condition $\beta_t Z_t^{-1} = s$ has the effect of making the capital-augmenting technology term constant. Second, the condition $g_{1-\beta} = -q^h$ has the effect of making the labor-augmenting technology term grow at a constant rate (recall that $\rho < 0$). In this way, although all technological advance is embodied in capital, aggregate production appears to have purely labor-augmenting technological progress, and we can satisfy the Uzawa Theorem (and match the Kaldor facts). Further intuition for balanced growth can be given in terms of the capital and labor requirements per unit of GDP.[7] A key distinction between this approach to a BGP and others in the literature is that, in equilibrium, the production function above does not become Cobb-Douglas. Rather, we continue to have an elasticity of substitution between capital and labor that is less than one.

Balanced Growth and Technology Limits.—The central feature driving the balanced growth path is the behavior of the two forms of technological progress. These technology pathways are also interesting compared to standard theories and intuitions. Namely, the technology indices, Z_t and β_t, are not growing at constant rates on the balanced growth path. Instead, they are approaching finite limits. Yet despite these limits, a balanced growth path can continue forever.

Specifically, balanced growth requires, via (BGP2), that $1 - \beta_t$ declines at a constant rate. That is, the automation limit occurs as $\beta_t \to 1$, and the share of nonautomated tasks $1 - \beta_t$ shrinks proportionally with time. Further, from (BGP1), balanced growth requires that the capital quality index Z_t follows the same dynamics as β_t. This implies that Z_t must also grow toward a limit along a balanced growth path. In particular, the limit of Z_t on a balanced growth path is $1/s$, as seen directly from (BGP1). A balanced growth path thus occurs not when β_t and Z_t grow at a constant rate; rather, it occurs when $1 - \beta_t$ and $1/s - Z_t$ decline at a constant rate.[8]

From labor's perspective, the share of labor being automated per unit of time is constant along a balanced growth path. As labor is allocated to a smaller measure of tasks, we have proportionally more labor per measure of these nonautomated tasks ($l_t(i) = L_t/(1 - \beta_t)$), which causes the output of nonautomated intermediates to grow proportionally as well. This feature follows Aghion, Jones, and Jones (2019), who consider the growth implications of proportional declines in the share of nonautomated tasks. However, the rise in β_t alone also raises the capital share of income. To achieve balanced growth, we must further consider the dynamics in Z_t. When Z_t rises, it acts to reduce the capital share of income, other things equal. If Z_t follows the same dynamics as β_t, then balanced growth is achieved (and when β_t and Z_t follow alternative dynamics, the economy experiences nonbalanced growth, as investigated in Section II).

[7] Balanced growth is characterized by a path where capital per unit of GDP is constant, while the labor requirement per unit of GDP is declining exponentially. Regarding capital, in the model the expanding share of tasks that are automated (β_t) will cause capital per unit of GDP to go up, but the increasing productivity of these capital inputs (Z_t) will cause capital per unit of GDP to go down. These contending forces allow aggregate capital to remain in constant ratio to GDP. Regarding labor, the exponentially declining share of tasks performed by labor means that labor can produce exponentially more of the remaining nonautomated tasks. Coupled with capital deepening as capital takes over the other tasks, the labor requirement per unit of GDP declines exponentially.

[8] This is the rate q^h (see (BGP2)).

These technology processes recall Zeno's Paradox. In such processes, one takes an infinite number of steps yet never completes a journey of finite length because each step is a fraction of the distance that remains. Here, even though the technology indices approach finite limits in this way, growth can continue forever. The model thus has a "limited innovation, limitless growth" feature. In a sense, one can be an extreme techno-pessimist and still see steady-state growth. While limits in technological progress are nonstandard in growth theory, such limits may also be appealing if one believes that there is some kind of fishing out process at work in innovation or if there are natural laws, like Carnot's efficiency maximum, that put finite limits on technological possibilities. Nonetheless, steady-state growth continues forever despite these limits, where the growth rate in Z_t and β_t fall along the BGP and limit to zero. Effectively, even if technological progress in the overall quality index of capital inputs and automation continually slows, there is continual and balanced growth through the use of machines.

We encapsulate these technology pathways on the balanced growth path in the following corollary.

COROLLARY 1 (Limited Innovation, Limitless Growth): *On the balanced growth path, the technology indices shrink at constant rates*, $g_{1-\beta_t} = g_{1/s-Z_t} = -q^h$, *but grow at shrinking rates*, $g_{\beta_t} = g_{Z_t} = \left[(1-\beta_t)/\beta_t\right]q^h$, *and approach finite limits*, $\beta_t \rightarrow 1$ *and* $Z_t \rightarrow 1/s$.

PROOF:

See Appendix.

The slowdown in the progress of the aggregate index Z_t on a BGP may seem restrictive or difficult to manage at a micro level. However, this BGP condition does not imply that any particular capital-embodied technologies face a slowdown, and, as we will see next, one can deploy a more standard micro-level quality ladder approach within this framework.

Balanced Growth and Task-Level Technological Advance.—The description of the balanced growth path has so far focused on aggregates and the BGP trajectories of β_t and Z_t. Here, we focus on the task-level productivities, $z_t(i)$, that constitute the aggregate technology index Z_t. Specifically, we examine task-level technology pathways that can produce BGP behavior in Z_t.

Generally, the evolution of Z_t can be understood as being driven by two forces: productivity advances on already-automated lines and the initial productivity of newly automated lines. In practice, there are many ways one can proceed at the task level so that a balanced growth path emerges. Using the definition (6), we can rewrite (BGP1) as

$$\int_0^{\beta_t} z_t(i)^{-1}\,di = s. \tag{25}$$

Thus, one can consider any innovation processes for which this is true. Natural processes of technological advance will see $z_t(i)$ increase with time on existing, automated lines. Meanwhile, newly automated technologies will add new initial productivity levels. The balanced growth path is possible when the productivities aggregate as in (25). To the extent that technology proceeds differently from (25), the economy will experience capital share dynamics and deviate from the BGP (see Section II).

As a specific innovation process, consider a vertical innovation model with proportional technology advances as seen in the endogenous growth literature. In particular, let an innovation on an existing automated line, $i \in [0, \beta_t]$, increase that line's productivity according to the process $[1 + \phi_t(i)]z_t(i)$, where in expectation $E_t[\phi_t(i)] = \phi$. Let such innovations occur with hazard rate q^v, where the superscript v denotes the "vertical" nature of such innovation. To satisfy (25), one then needs the new automation to enter as follows.

Case 1 (A Micro-innovation Process for Balanced Growth)**:** Let existing tasks increase their productivity by a proportional amount ϕ in expectation, with hazard rate q^v. Then a BGP will occur so long as newly automated tasks have productivity

$$z_t(\beta_t) = h(1 - \beta_t), \tag{26}$$

where $h \geq 1/(1 - s)$ and $s = q^h/(q^v \phi h)$.

PROOF:

See Appendix.

This corollary provides one set of sufficient conditions for innovation at the task level, allowing the Z_t index to evolve to produce the BGP. In particular, the corollary tells us how newly automated technologies draw their initial productivities, when existing automated tasks follow a quality ladder model. The parametric condition $h \geq 1/(1 - s)$ ensures that newly automated technologies are productive enough to be adopted given the equilibrium wage. The capital share in this technology process is $s = q^h/(q^v \phi h)$.

In this micro specification, the basic idea is that easier things are automated first. Tasks that are harder to automate (they are automated later) enter initially with lower productivity. Specifically, in (26), the initial quality of the newly automated technologies is falling along the growth path, tracking $1 - \beta_t$. To further understand this case, note that wages are rising on the growth path. As labor becomes an increasingly expensive production approach, lower-productivity automation technologies become increasingly worthwhile to adopt, and the technology adoption condition, (21), in particular tracks $1 - \beta_t$. This points to some deeper intuition for why the initial productivity of automated technology can fall on the growth path of the economy. The following case provides an example with productivity draws from Pareto distributions.

Case 2 (A Micro-innovation Process with Stochastic Step Sizes)**:** Let the hazard rate for successful vertical innovation be q^v with new productivity drawn from a Pareto distribution with shape parameter $\alpha^v > 1$. Similarly, let the hazard rate for successful horizontal innovation be q^h with initial productivity drawn from a Pareto distribution with shape parameter $\alpha^h > 1$. Then there is a balanced growth path.

PROOF:

See Appendix.

This case helps emphasize that, as wages rise, the threshold for implementation of an automated technology falls, which can naturally produce the declining average productivity of newly automated technologies with time. It further presents a candidate technology process with a more unified view of the horizontal and vertical dimensions, where balanced growth emerges when there are on average proportional advances over the technology adoption thresholds. In particular, the expected value of a draw from a Pareto distribution, conditional on beating some minimum threshold value, is proportional to that threshold value. The minimum threshold for vertical innovation is the current state of technology on that line, $z_t(i)$. The minimum threshold for a horizontal innovation is $z_t^{\min}$, which in turns tracks $1 - \beta_t$ on the balanced growth path. Thus, with Pareto distributions, in expectation vertical step sizes will be proportional steps over the existing $z_t(i)$, and horizontal steps will be proportional to $1 - \beta_t$, as in (26).

Finally, one can consider weaker sufficient conditions for a BGP, allowing for potentially substantial heterogeneity in vertical technological progress across technologies. One can then incorporate features like Moore's Law, where certain technologies may see much more rapid vertical technological advance than others and still produce balanced growth. This case is presented in the Appendix.

Overall, the task-level innovation approaches above provide sufficient conditions to satisfy (25) and generate a BGP in the exogenous growth model. Proportional vertical advances among already-automated technologies and declining productivity among newly automated technologies also set up a tractable approach for an endogenous growth model. We will return to endogenous growth in Section III to further analyze balanced growth.

Labor-Augmenting Technological Progress.—The model emphasizes that a BGP occurs, despite Uzawa, with purely capital-embodied technological advance. One can, however, also add labor-augmenting technological progress to the framework. We present such an extension in the Appendix and discuss some key intuition here. Specifically, one can replace the labor productivity term, A, with A_t in (5) and thereafter above. With standard, exponential growth in A_t, we have an additional force for growth in per capita income but otherwise no substantive effects on the equilibrium outcomes in the model, and notably, the capital share is unchanged.[9] Intuitively, the advance of labor productivity does not affect automation because the wage scales linearly in labor productivity, A_t; advances in labor productivity thus confer no cost advantage or disadvantage compared to using capital. More generally, as with Uzawa and traditional growth models, labor-augmenting technological progress is consistent with balanced growth. See Appendix for details.

[9] The growth rate is now $g = [(\rho - 1)/\rho]q^h + g_A$, while the capital share remains $s = \beta_t Z_t^{-1}$.

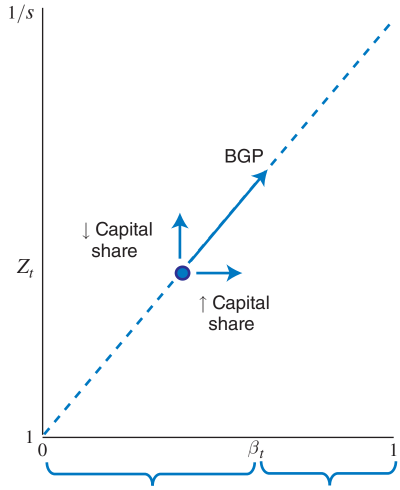

Figure 3. Technology Dynamics on and off the Balanced Growth Path

Distinctions from Prior Approaches.—The framework that emerges here is distinct from prior literature. From the BGP point of view, this paper is distinctive in that it can fully embody technological progress in capital equipment—despite the Uzawa Theorem's seeming prohibition on doing so.

By contrast, Acemoglu and Restrepo (2018, 2019a, 2019b, 2020) develop a task-based framework where the automation of existing tasks is counterbalanced by the creation of new tasks, all performed by labor. By adding new labor tasks, the share of automated tasks can remain constant, fixing the capital share. Those models further attach productivity gains directly to labor to allow balanced growth in non-Cobb-Douglas settings. Through the lens of the current framework, the Acemoglu and Restrepo BGP is akin to a path where Z_t and β_t are constant while A_t grows exponentially. Here, we allow a BGP where Z_t and β_t can grow and where A_t growth is allowed but is no longer needed. By allowing for vertical improvements in capital equipment in a non-Cobb-Douglas environment, the model emphasizes core features of technological progress, leads to a novel BGP, and provides additional novel applications and interpretations of macro phenomenon that we will discuss in Sections II and III.

Grossman, Oberfield and Simpson (2017, 2021) do allow for advances in capital productivity and show that this can be offset by increased education in the aggregate production function, leading to a BGP. That framework does not feature tasks or an automation dimension of capital advances, where capital can take over from labor, and it relies on increasing education as a key balancing force to create the BGP. In this paper, we elucidate key interplay between horizontal and vertical advances in capital and overcome Uzawa using capital alone.

Beyond the BGP, the current framework is distinctive in its micro roots by incorporating two forms of technological advance that both seem fundamental in economic history—both intensive and extensive advances in capital equipment (see the introduction). With these microfoundations, one can also speak to a wide variety of applications beyond the BGP and in a tractable way. These include factor share dynamics, long-run structural transformation, general-purpose technologies, and puzzles like Solow's Paradox. We turn to non-BGP features and applications next.

H. *Unbalanced Growth*

The model more generally develops analytic solutions for all the endogenous quantities and prices in the economy beyond balanced growth. The results can therefore speak to the relationship between technological change and economic dynamics, both on and off a balanced growth path. For example, the path of GDP is determined analytically in (19) in terms of exogenous variables and evolves according to the development of β_t and Z_t. We can encapsulate the equilibrium for more general sets of technology pathways (i.e., without assuming balanced growth) with a weak set of technology conditions as follows.

CONDITION 2: *Let the* $z_t(i)$ *be sufficiently large to satisfy the technology adoption condition* (21), *and let* $\omega - n > (1 - \theta)\bar{g}$ *satisfy the transversality condition.*

PROPOSITION 2 (Unbalanced Growth Path): *Under Condition 2, an equilibrium exists with a capital share* $s_{K_t} = \beta_t Z_t^{-1}$ *and explicitly determined paths of output, consumption, investment, the wage, the interest rate, and all other prices and quantities in the economy.*

PROOF:
See online Appendix.

Studying more general relationships between technology dynamics and economic outcomes will be the focus of Section II.

I. *Summary*

The key features of the model follow from having two margins of capital-embodied technological advance, creating a balance of forces that drive the trajectory of the economy. We have an automation process, with state variable β_t, featuring an extensive mode of advance as new tasks are automated. We separately have a capital-quality process, with state variable Z_t, encapsulating an intensive mode of advance at already-automated tasks. Thus, the model can have new capital goods (such as farming equipment, robots, or computers) taking over human tasks while also having improvement in the capital goods over time (better farming equipment, better robots, or better computers). In the tandem of these forces, balanced growth can emerge. In effect, having two margins of capital-embodied technological progress allows the model to overcome the tension between capital-embodied technological progress, a nonunitary elasticity of substitution, and balanced growth. In addition to satisfying the Uzawa Theorem and providing a balanced growth path, the model also provides closed-form solutions to study the economy off of a balanced growth path and thus engage a broad range of phenomena, which we turn to next.

II. Technology Dynamics and Economic Dynamics

The model allows for rich economic dynamics based in the two forms of technological progress. This section considers further applications and intuitions. We first show how Z_t and β_t dynamics may inform recent US economic growth and income share dynamics. We then consider structural change, showing how the economic transformation from agriculture and manufacturing to services can be built from differential progress in these two technology dimensions. We then apply the model to the concept of "general-purpose technologies" and engage Solow's Paradox, where extremely rapid technological progress (as in computing) can have limited macroeconomic effects.

A. *Income Shares and Growth Dynamics*

Consider first dynamics in the labor share of income and the growth rate. The model provides closed-form solutions for both outcomes outside a balanced growth path (see (17) and (19)). The dynamics are encapsulated in the following corollary.

COROLLARY 2 (Labor Share and Growth Dynamics): *The labor share is decreasing with time if $g_{\beta_t} > g_{Z_t}$, constant if $g_{\beta_t} = g_{Z_t}$, and increasing with time if $g_{\beta_t} < g_{Z_t}$. The growth rate in income per capita increases in g_{β_t} and g_{Z_t}.*

PROOF:
See online Appendix.

The model thus presents two technological stories for a declining labor share. Productivity gains at existing automated technologies raise the labor share, while further automation reduces the labor share (See Figure 3).

To the extent that there has been a concomitant growth slowdown, this can sharpen the technological picture. Taking the common description of the US economy as experiencing declines in both (i) the labor share and (ii) the growth rate, the model points toward a decline in g_{Z_t} as a parsimonious force that can deliver both phenomena at once. These dynamics, based in Z_t, further distinguish the framework from prior task-oriented approaches.

More subtly, while Z_t is growing on a balanced growth path, outside a balanced growth path, the index Z_t may actually decline in equilibrium as automation advances. This can lead to a sharp decline in the labor share. In particular, new, low-productivity automation technologies can drag down Z_t (the harmonic average of the capital productivities). Further, if innovation is becoming harder in the sense that vertical improvements in existing automated technologies are small, then there is little pushing up on the productivity index, and the newly automated technologies become the central force in driving the evolution of Z_t. Weak vertical innovation rates and weakly successful automation technologies can result in especially anemic gains from labor's perspective.[10]

[10] See also Acemoglu and Restrepo (2019a) for discussion of "so-so" technologies but without dynamics related to Z_t. Here, it is the productivity of the newly automated technologies themselves that is critical. More generally, these

We can formalize a "worst-case" (for labor) automation-led style of growth as follows, where output per capita grows at a constant rate through automation but wages stagnate.

COROLLARY 3 (Automation-Led Growth with Constant Wages): *Let automation proceed at some rate* $q^h > 0$*, where newly automated technologies have productivity level* $z_t^{\min}$*, the lowest level of productivity where they will still be adopted. Let prior automated technologies see no productivity improvement. Then wages remain constant. Income per capita grows at rate* q^h*, and the labor share falls at rate* q^h*. The technology index* Z_t *declines at rate* $q^h\left[(1 - Z_t)/\beta_t\right] < 0$.

PROOF:

See online Appendix.

This corollary presents a kind of weak but sustained growth in income per capita, with none of the gains going to labor. This result may be especially interesting as a technological context to consider recent dynamics in the US economy.

Application to US Growth and Income Share Series.—We now consider an application to US macroeconomic data, using standard data series for US labor productivity (output per hour) and the US labor share of income since 1950 (Bureau of Labor Statistics 2021a,b). See the top panel of Figure 4. While the recent productivity growth slowdown and the decline in the labor share, which appear in these series, are subjects of ongoing measurement debates, here we take these data series as given and see how the model would explain these patterns.

The bottom panel of Figure 4 presents the estimated technology paths. Specifically, one may pin down β_t and Z_t each period using the output and labor share outcomes, (18) and (19).[11] These paths are presented as $1 - \hat{\beta}_t$ and $1/s - \hat{Z}_t$, which is useful for seeing balanced growth behavior and deviations from this behavior.

The estimation results suggest striking shifts in the nature of technological progress. First is an era of relative stability. Horizontal and vertical innovation proceed at broadly similar rates from 1950 to 1990, consistent with broadly balanced growth.[12] Second, starting around the mid-1990s, we see episodes of decoupled technological progress and unbalanced growth, which continue until around 2010. Third, after 2010, we again see parallel but slower progress in both forms of innovation. This most recent period is consistent with innovation getting harder in a general sense while also maintaining a relatively stable labor share. Of course, the labor share is structurally lower after 2010, which follows from highly unusual behavior from 2000 to 2010. This period follows the broad logic of Corollary 3: automation (in fact, a burst of automation) but a reversal in the capital productivity index, consistent

results are unusual because the technology index Z_t can actually decline in equilibrium. This is very different from most growth models, where factor-augmenting technology terms normally only go up as technology advances.

[11] The Appendix provides further details on this estimation. Note that with the two technology indices, the model can fit both the output and labor share outcomes exactly.

[12] The 1970s-era slowdown in labor productivity growth is reflected in slowdowns on both the vertical and horizontal technology dimensions, so that, while the growth rate declined, the labor share of income remained relatively steady.

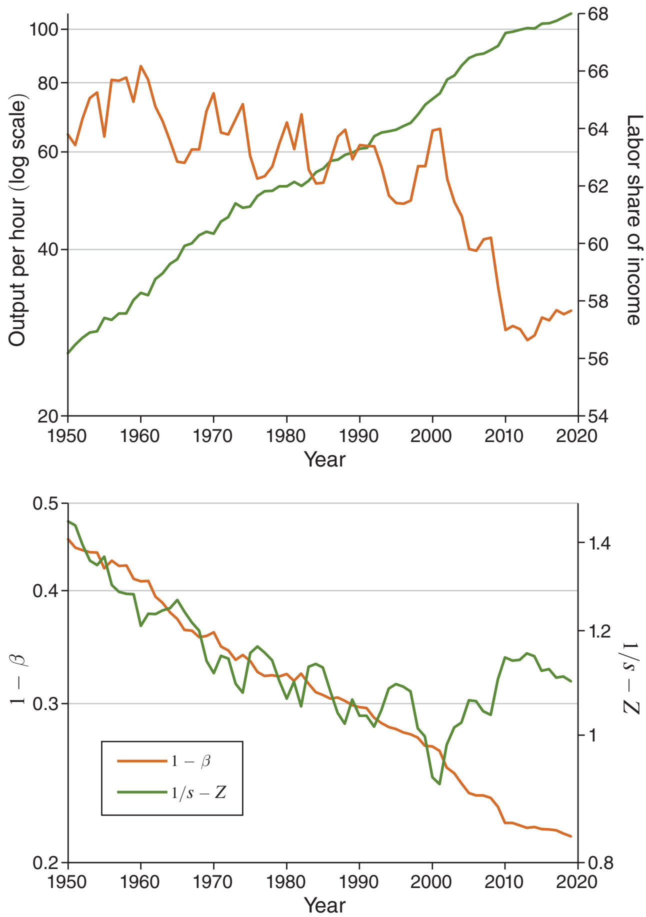

Figure 4. US Macro Dynamics and the Two Technologies

Notes: Top: Output per hour and the labor share of income. Bottom: Estimated paths of automation and the capital productivity index.

with the entrance of new automation technologies that are not much lower cost than labor. The result is anemic growth, and especially anemic gains for labor.

B. *Structural Change*

The model can also be applied to structural change. The distinguishing feature is again the interplay between Z_t and β_t but now applied to sectors. For example, agricultural technologies advance by automating labor tasks and by improving at previously automated tasks, both central margins at a micro level (see Figure 1) that lead to distinctive interpretations of sectoral dynamics.[13]

[13] Structural change models emphasize demand-side forces (e.g., Kongsamut, Rebelo, and Xie 2001; Buera and Kaboski 2012; Herrendorf, Rogerson, and Valentinyi 2013; Matsuyama 2019; Comin, Lashkari, and Mestieri 2021) and supply-side forces grounded in technological progress (e.g., Baumol 1967; Acemoglu and Guerrieri

To formalize sector-level analysis in a straightforward manner, one can label subsets of tasks as representing different sectors. Specifically, let there be J sectors indexed $j \in \{1, \ldots, J\}$. Let each sector have a measure $u_t^j \leq 1$ of tasks, which divide up all the tasks in the economy so that $\sum_j u_t^j = 1$. Further, let the measure of automated tasks in sector j be $\beta_t^j \leq u_t^j$, which sum to the overall automated task share, $\sum_j \beta_t^j = \beta_t$. Finally, let each sector have a technology index Z_t^j, which is the harmonic average of the $z_t(i)$ in that sector.

By summing up the value of output within a sector, it is straightforward to write

$$\Phi^j = \beta_t^j \left(Z_t^j\right)^{-1} + s_{L_t} \frac{u_t^j - \beta_t^j}{1 - \beta_t}, \tag{27}$$

$$s_{L_t}^j = 1 - \beta_t^j \left(Z_t^j\right)^{-1} \frac{1}{\Phi^j}, \tag{28}$$

where Φ^j is the GDP share of the sector and $s_{L_t}^j$ is the labor share of income within the sector.

With these expressions, we can consider the influence of both forms of technological advance.[14]

COROLLARY 4 (Structural Change)*: Holding other sector technology levels fixed, the GDP share of a sector will decline with its automation level, β_t^j, or capital productivity level, Z_t^j. The labor share of income within the sector will decrease in the automation level, β_t^j, but increase in the capital productivity level, Z_t^j.*

PROOF:

See online Appendix.

These findings present a form of Baumol's cost disease, where technological advance of either type causes the sector's GDP share to decline. By contrast, the type of technological advance has opposing implications for labor shares within the sector. Related logic applies when the elasticity of substitution between tasks differs from that between sectors, so long as both the "inner" and "outer" elasticities of substitution are less than one. The key point is that the offsetting forces mean that one can locate the nature of technological change distinctively between extensive and intensive technological advance. We consider this application next.

Application to US Structural Transformation.—With these sector-level results and intuitions, we again consider an application of the model to US macroeconomic data. The top panel of Figure 5 presents data for three sectors: manufacturing, agriculture, and other private sector (which is mostly comprised of services), presenting

2008; Acemoglu and Restrepo 2019a). This model sits on the supply side. With the elasticity of substitution less than one, the approach engages a Baumol cost disease perspective (Baumol 1967) but with the capacity to unpack and estimate sector-level technological change on the extensive (β_t) and intensive (Z_t) dimensions.

[14] On the growth path of the economy, technological advance may naturally be occurring in all sectors simultaneously. A variant of this corollary can emphasize the relative evolution of sectoral technological indices. That formulation produces similar results and intuition and is provided in the online Appendix as Corollary 4a.

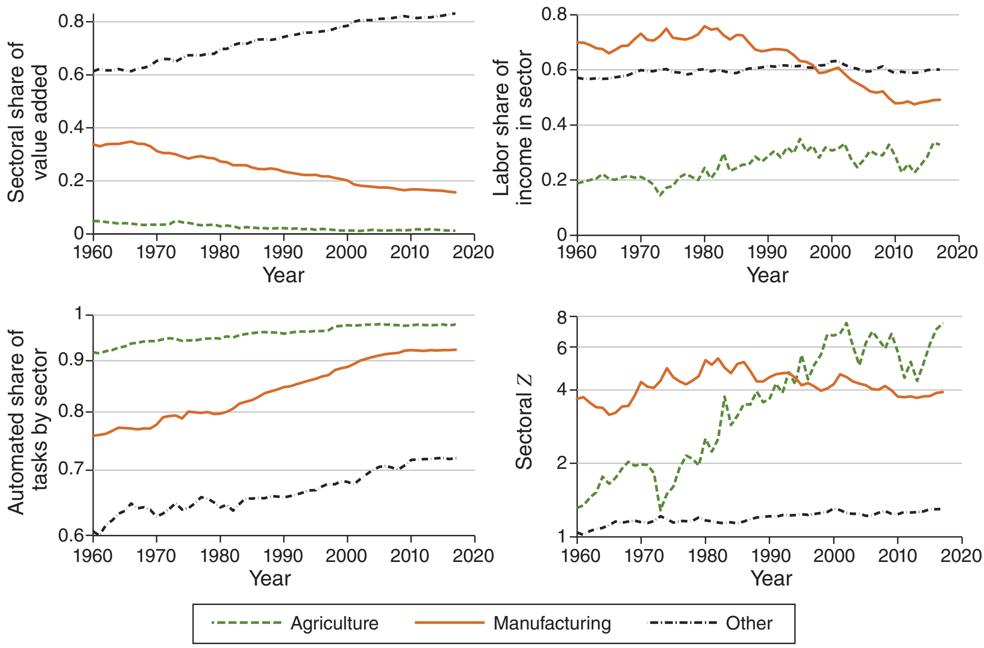

Figure 5. US Structural Change and Technological Change

Notes: Top: Raw data for sectoral GDP shares (left) and labor shares (right). Bottom: Estimated share of automated tasks within sector (left) and capital productivity index for sector (right). Sectoral data come from Mendieta-Munoz, Rada, and Arnim (2020).

GDP shares and the labor shares of income within each sector. For data, we use Mendieta-Munoz, Rada, and Arnim (2020).[15]

The top panel of Figure 5 shows a familiar picture of structural change. Agriculture (already a low share of GDP) and especially manufacturing show declining GDP shares with time, while the GDP share elsewhere rises. Further, manufacturing exhibits a sharp decline in the labor share of income over the 1980–2010 period. The labor share in services is steadier, and in agriculture it is rising.

To consider these patterns through the lens of the model, we can describe each sector by the technology vector $\left\{u_t^j, \beta_t^j, Z_t^j\right\}$, representing the sector's share of tasks, automation rate, and capital-productivity index, where all sectors aggregate to the economy-wide measures $\{1, \beta_t, Z_t\}$. Using (27) and (28), we can then use data series for the sectoral GDP share and within-sector labor share of income to estimate $\hat{\beta}_t^j$ and $\hat{Z}_t^j$.[16] The bottom panel of Figure 5 presents, for each sector, the estimated paths of automation and capital productivity. Consider automation first. We see a durable rank ordering—agriculture is the most automated, followed by

[15] Mendieta-Munoz, Rada, and Arnim (2020) provide value added output and labor shares of income for 14 sectors. Their approach drops the public sector and housing sector. Here, we study (i) manufacturing; (ii) agriculture, forestry, and fishing; and (iii) other, which includes arts, entertainment and recreation; construction; education, health and social services; finance and insurance; information; mining; retail trade and wholesale trade; transportation and warehousing; professional and business services; utilities; and other services.

[16] For these estimates, we determine the tasks shares using SIC and NAICS industrial coding schemes, as discussed in the online Appendix.

manufacturing, followed by services—even as each sector is becoming more automated with time. Notably, manufacturing is catching up to the high automation share in agriculture, and automation proceeds especially quickly in manufacturing over the 1980–2010 period. This burst of automation is consistent with the declining labor share in manufacturing over this period. Examining the capital productivity indices, we see that Z_t^{agr} exhibits much greater advance than the other sectors. This acts toward reducing the GDP share of agriculture and increasing the labor share in agriculture. By contrast, the advancing automation in manufacturing comes with weak and even retrograde movement in Z_t^{man} after 1980, which can further inform the declining labor share of income in this sector. Meanwhile, we see anemic growth in Z_t^{other}. Expanding service automation has not been coupled with new technologies that are substantially more productive than labor. The rising GDP share of services follows in part by this relatively anemic progress. Of course, as services become the increasingly dominant share of the economy, technological progress in this sector increasingly dominates the path of the economy overall. The relatively weak progress in this sector historically and recently thus suggests potentially enduring growth challenges.

C. *General-Purpose Technologies and Computing*

Finally, consider a general-purpose technology (GPT)—for example, computers. Here, a type of capital input takes on a widening variety of tasks, making it "general-purpose." The model provides a simple way to understand the role of a GPT in the economy, again as the interplay of horizontal and vertical advance.

Specifically, we can define a general-purpose technology as a type of capital input that automates a measure β_t^{GPT} of tasks. This measure will increase as the GPT takes over more tasks.[17] But then there are also advances in productivity, $z_t(i)$, for these general-purpose capital inputs. These productivity gains will generally occur in heterogeneous ways across tasks. For example, the productivity at some computer-performed tasks (e.g., floating-point operations per second) has risen at remarkable rates, while computer automation at other tasks (e.g., voice transcription) has shown more modest productivity improvements. The productivity index across the GPT tasks, Z_t^{GPT}, can then grow (though determined relatively strongly by its lower-productivity tasks) as the GPT simultaneously extends its automation footprint. The GPT's share of GDP is then simply,[18]

$$\Phi_t^{GPT} = \frac{\beta_t^{GPT}}{Z_t^{GPT}}. \tag{29}$$

For example, here, computers grow as a share of GDP as they take over more tasks, but the rising productivity of computers reduces their share of GDP.

[17] This can be new automation taking over from labor. For example, the word "computer" once referred to a worker type but now refers to a machine (see Figure 1). The general-purpose capital input might also take over from other capital inputs as a vertical advance; for example, computer storage can take over from printed books.

[18] This follows directly by aggregating the capital inputs in (15) over a measure β_t^{GPT} of tasks performed by the GPT. Alternatively, using the lens of the structural change analysis above, we can define a tuple $\{u_t^{GPT}, \beta_t^{GPT}, Z_t^{GPT}\}$ where $u_t^{GPT} = \beta_t^{GPT}$ is the evolving measure of tasks performed by the GPT and use (27).

Computers are worth further consideration because of their broad use and because Moore's Law presents a remarkable and continuous improvement in certain productivity metrics. In fact, the growth slowdown often seems in tension with the extraordinary progress in computing and its broad applications—Solow's Paradox (Solow 1987; David 1990). As a final result, consider then the limited power of seemingly amazing technologies in this model. In particular, recall from (6) that Z_t is the harmonic average of the $z_t(i)$. Recall also that harmonic averages are heavily weighted toward their smallest values. For example, the harmonic average of a finite number and infinity is twice the finite number. Thus, in this model, the productivity at some tasks can advance enormously (e.g., via Moore's Law) and even go to infinity. However, Z_t (the harmonic average) may not increase substantially. This feature follows from the less-than-unitary elasticity of substitution (see also Aghion et al. 2019). Rather than having the highest-productivity sectors take over the economy, these sectors become smaller shares of the economy.

Formally, we can encapsulate the limited effect of extreme productivity at any particular measure of automated tasks, as follows.

COROLLARY 5 (Extreme Technological Advance): *Let a fraction α of the automated tasks have the same distribution of $z_t(i)$ as the other automated tasks. Holding other technologies constant, take $z_t(i) \rightarrow \infty$ for this fraction α of automated tasks. The capital share will decline by α percent. Income per capita will increase by* $\Delta \ln(y_t) = -(1/\rho)\ln\left(1 + \alpha\left(s_{K_t}/s_{L_t}\right)\right)$.

PROOF:

See online Appendix.

So, for example, if $\alpha = 10\%$ of automated tasks, and these suddenly became infinitely productive, then a capital share of, say, $s_K = 0.40$ would fall to 0.36. Taking an elasticity of substitution between capital and labor of 0.5 ($\rho = -1$), which is a typical value in the literature, then income per capita would increase by 6.5 percent. Thus, even extreme (infinite!) advances in productivity for substantive shares of the economy's tasks, which would appear dramatic for the affected sectors, would have substantially muted macro effects. This suggests a possible view on the limited effects of computing advances and Moore's Law at an aggregate level.

III. Endogenous Growth Model

The baseline model of Section I can solve the challenge of the Uzawa Growth Theorem while also engaging rich industrial and macroeconomic dynamics. It is less clear, however, why a balanced growth path may tend to emerge amid the tug-of-war between automation and capital quality improvements. And, more generally, the pathways of automation and capital productivity improvements are not determined inside the baseline model. In this section we extend the baseline model to allow for endogenous growth.

To model endogenous technological progress, we follow the standard setup (Romer 1990; Aghion and Howitt 1992) of introducing profits into the intermediate goods sector and letting these profits pay for R&D. Thus, we will edit the baseline growth model by (i) introducing market power on intermediate capital goods and (ii) introducing knowledge production functions that relate innovation outcomes to input costs of R&D. A point of difference with standard endogenous growth models is that we now have both vertical innovations, increasing $z_t(i)$, and horizontal innovations, increasing β_t. For simplicity, we set $L_t = L$, a constant.

A. *Capital Good Producers*

As before, producers of final goods and nonautomated intermediate goods remain competitive. However, producers of automated goods now have market power. We follow the vertical growth literature (Aghion and Howitt 1992), where the firm with the leading technology is a monopolist (supported, e.g., by an infinitely lived patent) but faces a competitive fringe that accesses a lower-productivity vintage of technology. While we will closely follow this standard vertical approach, some distinctions will occur here because $\rho < 0$.[19] Specifically, the limit price imposed by the competitive fringe always binds as the profit-maximizing price set by the leading firm. Thus, we replace the equilibrium price (13) in the baseline model with

$$p_t(i) = \psi z_t^e(i)^{\frac{1-\rho}{\rho}}, \quad i \in [0, \beta_t], \tag{30}$$

where $z_t^e(i)$ is the technology accessed by the competitive fringe.

Innovation steps are proportional increases of size ϕ in the prior technology level. We assume the competitive fringe accesses a level of technology

$$z_t^e(i) = \gamma z_t(i),$$

where $\gamma \in [1/(1+\phi), 1]$.[20] This competitive fringe pins down the equilibrium price

$$p_t(i) = \psi [\gamma z_t(i)]^{\frac{1-\rho}{\rho}}, \quad i \in [0, \beta_t], \tag{31}$$

which is the limit price used by the monopolist firm with the leading technology. Given this price, the demand for the good $y_t(i)$ is, using (11),

$$y_t(i) = \upsilon^{-1} \gamma^{-\frac{1}{\rho}} z_t(i)^{-\frac{1}{\rho}} Y_t, \quad i \in [0, \beta_t]. \tag{32}$$

The innovator will fulfill this quantity demanded at the limit price, but using their new technology. This determines the scale of the new innovator's production, defining the $x_t(i)$. Setting demand equal to supply for $y_t(i)$, we have

$$x_t(i) = \upsilon^{-1} \gamma^{-\frac{1}{\rho}} \frac{Y_t}{z_t(i)}, \quad i \in [0, \beta_t]. \tag{33}$$

[19] This means that the final good producers' demand, given by (11), differs from the traditional Cobb-Douglas case.

[20] Tying the competitive fringe's technology to exactly the prior technology vintage, $\gamma = 1/(1+\phi)$ is a potential simplification. Here, we decouple the markup from the technology step size so that market power and technology step size may be distinct forces. We consider the competitive fringe technology to be in the interval $\gamma \in [1/(1+\phi), 1]$, with the idea that the fringe imitates the leading technology to some extent.

And now there are profits. The flow profit for the new innovator is

$$\pi_t^v(i) = \mu \frac{Y_t}{z_t(i)}, \tag{34}$$

where we define $\mu = \gamma^{-1} - \gamma^{-1/\rho}$, capturing the limit-price (markup) effect.

B. *Innovation Process*

Resources are now devoted endogenously to advance R&D. The resource constraint of the economy is

$$C_t + I_t + D_t = Y_t,$$

where the new term, D_t, represents the total expenditure on R&D.

There are two different types of innovations, corresponding to the margins of technology in the model. First, there are vertical innovations, where innovating firms follow a quality ladder, improving technologies that have already been automated. Second, there are horizontal innovations, where innovating firms automate new tasks. We consider each in turn.

Vertical Innovation.—An intermediate firm may invest in vertical research in a (stochastic) attempt to raise a given productivity, $z_t(i)$, by a proportional amount ϕ. The flow value of profits from such an innovation is given by (34). While the technology level, $z_t(i)$, is fixed for a given vintage and patent, this flow profit is growing as the scale of the market grows, i.e., at the growth rate of the economy, g_t. The firm will lose these profits through creative destruction when another firm innovates along this line, which occurs with arrival rate $q_t^v(i)$. Defining the present value of an innovation as $V_t(i)$, we have the Bellman equation,

$$r_t V_t(i) - \dot{V}_t(i) = \pi_t^v(i) - q_t^v(i) V_t(i). \tag{35}$$

We will examine the balanced growth path equilibrium where g_t, r_t, and $q_t^v(i)$ are all constants, and the value of an innovation along the BGP can thus be written

$$V_t(i) = \frac{\pi_t^v(i)}{r + q^v - g} = \frac{\mu}{r + q^v - g} \frac{Y_t}{z_t(i)}. \tag{36}$$

The value of an innovation is a function of the task's market size. The market size for the task is increasing on the growth path as the overall economy expands. However, the intermediate good's market size is decreasing when its productivity rises.[21]

[21] Note that the latter effect is only operable across rungs of the quality ladder—the $z_t(i)$ is fixed from the perspective of the leading firm.

Entrant firms perform R&D with increased investment raising the arrival rate of innovation.[22] We specify a lab equipment model that features diminishing returns to effort but positive intertemporal spillovers on a given line. Namely, investing $d_t^v(i)$ in R&D at task i will generate an arrival rate of new ideas of

$$q_t^v(i) = \xi^v \left[\frac{z_t(i) d_t^v(i)}{Y_t}\right]^{\alpha}, \tag{37}$$

where ξ^v is a measure of how easy innovation is in the vertical direction and $\alpha < 1$.[23]

The zero-profit entry condition in R&D implies that

$$q_t^v(i) V_t(i) = d^v(i),$$

so that the expected value of R&D effort is equated to its costs in equilibrium. Given the R&D technology, (37), and the value of an innovation on the BGP, (36), this equilibrium entry condition on the balanced growth path becomes

$$\frac{\xi^v \mu}{r + q^v - g} = \left[\frac{z_t(i) d_t^v(i)}{Y_t}\right]^{1-\alpha}, \quad i \in [0, \beta_t). \tag{38}$$

On a BGP, it follows that q^v is a constant. In particular, from (38), the ratio $z_t(i) d_t^v(i)/Y_t$ is a constant on a BGP, so that more advanced lines (which have smaller shares of GDP and hence smaller market size) attract less effort in equilibrium. From (37), this leads to a constant hazard rate of vertical advance across automated lines.

Horizontal Innovation.—The other innovation margin is horizontal, where innovators seek to automate currently nonautomated sectors. We allow R&D to be conducted on any currently nonautomated line. Success occurs with arrival rate q_t^h. Further, a new automation technology must come with some initial technology level. For the newly automated task at time t, we assume that the initial automation quality follows the process

$$z_t^h = h(1 - \beta_t) \tag{39}$$

for some constant h. This assumption (which is familiar from Section IG) says that the initial automation quality is decreasing over time. This declining initial productivity feature is natural to the extent that lower-productivity automation technologies become worthwhile to implement as wages rise along the balanced growth

[22] Following standard vertical models (Aghion and Howitt 1992), entrant firms do all the R&D and seek to displace the incumbent firm.

[23] We specify a lab equipment model rather than a labor input model, as it is slightly more tractable. In the lab equipment model, expenditure on R&D is normalized by GDP in (37). This will lead to a constant GDP share being spent on R&D. This specification is similar to relying instead on R&D labor, $l_t^v(i)$, and the knowledge production function $q_t^v(i) = \xi^v[z_t(i) l_t^v(i)]^{\alpha}$. With labor paid the prevailing wage, this formulation also produces a constant R&D expenditure share of GDP and a balanced growth path. This variation is available from the authors upon request.

path. The linearity in $1 - \beta_t$ can also be natural noting that the threshold for technology adoption is itself linear in $1 - \beta_t$.[24]

Once a horizontal research investment succeeds, the newly automated task becomes a vertical line, and further innovation proceeds in the same vertical manner as for previously automated tasks, described above. Specifically, we assume there is a markup based on a technology advantage γ, which defines the flow profits from this innovation as in (34).[25] The present value of this new innovation, $V_t(i)$, will then again be as in (35) and (36). Creative destruction occurs when further, vertical R&D improves on this newly automated technology.

On the cost side, the horizontal knowledge production function will follow a similar structure of R&D costs as seen vertically, in (37), except that we allow the cost parameter to differ. Specifically, horizontal R&D investment, $d_t^h(i)$, which targets a nonautomated line, $i \in (\beta_t, 1]$, will generate an arrival rate for a new automation of

$$q_t^h(i) = \xi^h \left[\frac{z_t^h d_t^h(i)}{Y_t}\right]^{\alpha}, \tag{40}$$

where ξ^h is a measure of how easy innovation is in the horizontal direction. The zero-profit entry condition is then

$$q^h(i) V_t(i) = d_t^h(i).$$

On the balanced growth path, this entry condition becomes[26]

$$\frac{\xi^h \mu}{r + q^v - g} = \left[\frac{z_t^h d_t^h(i)}{Y_t}\right]^{1-\alpha}, \quad i \in (\beta_t, 1]. \tag{41}$$

On the BGP, it follows that q^h is a constant. As with the vertical logic above, from (41), the ratio $z_t^h d_t^h(i)/Y_t$ is a constant on a BGP. From (40), this leads to a constant hazard rate of automation among the nonautomated lines.[27]

We can now collect several results for R&D in the following lemma.

LEMMA 1: *The vertical and horizontal hazard rates of innovation*, q^v *and* q^h, *are both constants on a BGP and have the ratio*

$$\frac{q^h}{q^v} = \left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}. \tag{42}$$

[24] See Section IG. We can assume that the initial productivity is deterministic according to (39), as in Case 1 in the exogenous growth model. One can also consider a stochastic version, as in Case 2. With a Pareto distribution for initial productivity draws, the expected initial productivity level will be a constant proportion above the technology adoption threshold, $z_t^{\min}$. This will directly produce (39) in expectation and also guarantee that the initial technology level is above the adoption threshold.

[25] That is, we assume that the newly automated production tasks engender a competitive fringe that can access a technology a proportion γ worse than the newly automated technology. This assumption enhances tractability by creating symmetry with the vertical case but is not essential.

[26] Note that horizontal innovators attempt to innovate across the measure of nonautomated lines. If successful, the productivity of that line is given by (39). That is, we are treating all nonautomated lines as symmetric, and for notational simplicity, we are implicitly re-indexing a successfully automated task to stand at $i = \beta_t$.

[27] Note that q^v appears in (41) because, once initially automated, the expected value of the line depends on the rate of being replaced.

Aggregate R&D expenditure in the vertical direction and in the horizontal direction are both constant shares of GDP.

PROOF:

See online Appendix.

The relative innovation rate in the horizontal and vertical dimensions (q^h/q^v) is thus determined by the relative ease of innovating in these directions (ξ^h/ξ^v) and the degree of diminishing returns (α) when R&D effort crowds into one research avenue. With these innovation rates, together with the size of vertical steps, ϕ, and the corresponding "size" parameter for a horizontal step, h, we can determine the endogenous evolution of the technology indices, β_t and Z_t, and characterize the balanced growth path. To do so, we will first determine the economic aggregates and then return to the conditions that determine the BGP.

C. *Aggregates*

We calculate the capital stock, and the capital share of income, by summing up across the automated intermediate inputs using (33). The capital share of income is

$$s_{K_t} = \frac{\psi X_t}{Y_t} = \gamma^{-\frac{1}{\rho}} \beta_t Z_t^{-1}. \tag{43}$$

Comparing this outcome to its value in the exogenous growth model, (17), we see that less capital is used for a given technological state. This is the usual result in endogenous growth, where market power in this sector and the consequent markup has reduced its output and thus its input demand.

The aggregate profit is calculated by summing up profits across the automated lines using (34),[28]

$$\frac{\Pi_t}{Y_t} = \mu \beta_t Z_t^{-1}. \tag{44}$$

The labor income is the remaining part of the output,

$$s_{L_t} = 1 - \gamma^{-1} \beta_t Z_t^{-1}. \tag{45}$$

Aggregating the intermediate outputs, we find a similar result to the exogenous growth model. The difference is that monopoly power reduces GDP compared to the exogenous case with fully competitive markets. Adding up intermediate outputs and prices, aggregate GDP is[29]

$$Y_t = \frac{\upsilon A L (1 - \beta_t)^{\frac{1-\rho}{\rho}}}{\left(1 - \gamma^{-1} \beta_t Z_t^{-1}\right)^{1/\rho}}. \tag{46}$$

[28] Recall that nonautomated lines are produced competitively using labor.

[29] This form is familiar from the exogenous growth case. The difference is the markup parameter, γ, which acts to reduce GDP.

D. *The Endogenous Balanced Growth Path*

As with the exogenous growth model, we need the same two macro-level conditions for the economy to present a balanced growth path. First, we require $\beta_t Z_t^{-1} = s$, for some constant s, as before (BGP1). This condition produces a constant capital share, profit share, and labor share (see (43), (44), and (45)). Second, with a constant capital share, we see that income per capita in (46) will grow at $g = [(\rho - 1)/\rho] g_{1-\beta_t}$. Balanced growth thus requires that $g_{1-\beta_t} = -q_t^h$, a constant (BGP2).

We can now confirm that the endogenous innovation processes will meet these conditions. First, consider the horizontal innovation sector. We know via Lemma 1 that q^h is constant with time in equilibrium. Thus the horizontal innovation sector provides $g_{1-\beta_t} = -q_t^h$, a constant. This directly satisfies (BGP2).

Second, consider the vertical innovation sector. We have a step size of ϕ on each line and, again per Lemma 1, a constant rate of innovation, q^v, on the BGP. Further, newly automated technologies are drawn according to (39). It therefore follows that the ratio of the technology indices on the BGP is pinned down as[30]

$$\beta_t Z_t^{-1} = \frac{1}{h\phi}\left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}. \tag{47}$$

The technology indices β_t and Z_t thus evolve in a constant ratio, satisfying (BGP1).

This constant ratio in turn implies that the capital, profit, and labor shares of income will be constant on the BGP (see (43), (44), and (45)). The labor share specifically is

$$s^L = 1 - \frac{\gamma^{-1}}{h\phi}\left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}.$$

Thus, not only are the income shares constant with this innovation model, producing a balanced growth path, but they can be expressed in a simple form based on the parameters of the vertical and horizontal knowledge production functions and the markup parameter.

Finally, we can confirm that these newly automated technologies are in fact adopted—that is, that they are more cost-effective than using labor. This requires that the newly automated technologies be sufficiently productive initially.[31] This parametric condition is readily verified as follows, which we assume holds.

LEMMA 2: *Automated technologies are used on the BGP if* $h \geq \gamma^{-1}(1/\phi) \times (\xi^h/\xi^v)^{1/(1-\alpha)}$.

PROOF:

See online Appendix.

[30]This follows by combining the process of new productivity draws, (39), the condition (26) for newly automated technologies to maintain a BGP, and the relative rates of horizontal and vertical innovation (42).

[31]Noting that wages are rising on the growth path, being productive enough to be adopted when it is initially automated guarantees that the technology remains productive enough to continue using over time since labor becomes increasingly expensive.

This condition is the endogenous growth analog to (21) in the exogenous growth model. It further guarantees that the capital share in (47) is less than one.

With these results, we have determined the income shares in terms of the exogenous variables and now turn to the growth rate itself. For the general case, we have a system of four equations that determine four endogenous variables $\{q^v, q^h, r, g\}$. These four equations govern the vertical innovation rate (from (37) and (38)), the relationship between the vertical and horizontal innovation rates (42), the relationship between the growth rate and horizontal innovation rate (23), and the Euler equation (8), which we collect here:

$$\begin{cases} q^v = \xi^{v\frac{1}{\alpha}} \mu q^{v\frac{\alpha-1}{\alpha}} - (r-g), & \text{Vertical Innovation;} \\ \dfrac{q^h}{q^v} = \left(\dfrac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}, & \text{V\&H Relationship;} \\ g = \dfrac{1}{\theta}(r-\omega), & \text{Euler Equation;} \\ g = \dfrac{\rho-1}{\rho} q^h, & \text{Growth Rate.} \end{cases}$$

To study properties of the growth path, we focus on the case where $\theta \geq 1$.[32]

PROPOSITION 3 (Existence and Uniqueness): *The balanced growth path exists and is unique if* $\theta \geq 1$.

PROOF:

See online Appendix.

The model thus produces a balanced growth path, emerging endogenously from R&D effort on the vertical and horizontal dimensions. A primary intuition for the balanced result follows from the diminishing returns to R&D effort at a point in time on a given line, which we can think of as a crowding or duplication externality among competing R&D firms. This force acts to spread R&D effort out across lines, with the balance of vertical and horizontal progress depending on the knowledge production function parameters. This diminishing returns force also allows the equilibrium to move away from requiring knife-edge relationships among the knowledge production function parameters, and it creates conceptual degrees of freedom in the model, separating automation forces from capital productivity improvement forces in informing macroeconomic outcomes.

E. *Growth and the Labor Share of Income*

Tracking the two dimensions of technological progress, we can return to consider the apparent growth slowdown and decline in the labor share. As usual with an

[32]Empirical evidence commonly suggests the case where $\theta \geq 1$. See, for example, Campbell (2003) and Vissing-Jorgensen (2002). The model can also be considered where $\theta < 1$, with some further parametric restrictions, but this case is analytically more complex in addition to being less salient empirically.

endogenous growth model, we think of dynamics as a shift between two different balanced growth paths, where we have moved from a relatively high-growth and high–labor share equilibrium to a relatively low-growth and low–labor share equilibrium. With the capital share (and hence labor share) given explicitly by (47) and the growth rate determined implicitly in the four-equation system above, we can now consider the implications of changing parameters of the model. Key comparative statics are collected here.

PROPOSITION 4 (Endogenous Growth): *The labor share on a balanced growth path increases with* ξ^v, h, ϕ, *and* γ *and decreases with* ξ^h. *The growth rate on a balanced growth path increases with* ξ^h *and* μ, *decreases with* ξ^v, *and is unchanging in* h.

PROOF:

See online Appendix.

In light of these results, consider the apparent recent downshift in both the growth rate and the labor share (e.g., Gordon 2016; Elsby, Hobijn, and Sahin 2013; Karabarbounis and Neiman 2014). First, consider an increasing difficulty in innovation, and let's assume this happens on all dimensions. Specifically, let the ease of innovation, ξ^h and ξ^v, decline proportionally in both horizontal and vertical dimensions, and let the size of innovations, h and ϕ, also decline. If "innovation has become harder," both the growth rate and the labor share will decline. The decline in growth is intuitive when innovation gets harder. The decline in the labor share follows because advancing capital technology has become more difficult. With a less than unitary elasticity of substitution, a lower productivity capital stock makes it relatively costly, and it is cheap capital that supports the labor share.[33]

Second, one can look at more precise dimensions of knowledge production. First, consider the automation dimension. Here, we can think of automation as becoming harder, in two senses. We could imagine that the ease of discovering automation goes down (ξ^h decreases) and the productivity of the new automation becomes worse (h declines). First, the decline in h acts to pull down the labor share. Second, a decline in ξ^h will cause the growth rate to decline.[34] For example, if we think that recent automation has been barely good enough to replace labor (e.g., replacing customer service workers or secretarial workers with automated systems, but these automated systems perform poorly), this low automation quality will cause the labor share to fall. What is bad for labor share of income is being replaced by machines that aren't very good, so that there is little productivity gain to offset the displacement effect of increased automation.[35]

[33] This is the opposite of how one thinks about capital productivity and income shares with an elasticity of substitution greater than one (e.g., Karabarbounis and Neiman 2014). With an elasticity of substitution less than one, we can instead link a declining labor share and declining productivity growth in one frame.

[34] For the labor share to decline on net, we would need the percentage decrease in h to exceed $1/(1-\alpha)$ times the percentage decrease in ξ^h (see (47)).

[35] The issue of "so-so" technology here is distinct from Acemoglu and Restrepo (2018, 2019a), where productivity gains are embodied in labor. Here, the "so-so" technology is embodied in the automation; namely, we adopt a new automation technology, replacing labor, and this new type of capital equipment isn't very good.

Third, consider the vertical technology dimension. Think of innovation becoming harder in terms of vertical step sizes becoming smaller. To focus purely on technology, take the natural idea that a smaller vertical step size (ϕ) will lead to a closer distance between the competitive fringe and the frontier technology (γ). Then a decline in the vertical step size will reduce the growth rate and increase the capital share. A decline in the vertical technology step size can then provide an especially parsimonious force for both the macroeconomic effects. Interestingly, a smaller vertical step size will also raise the profit share of GDP in the economy.[36] An increasing profit share has also been seen in some recent evidence (e.g., Barkai 2020). These results are grounded in capital-embodied technology gains and an elasticity of substitution less than one, distinct from prior growth models.

Finally, consider a political economy dimension, focusing on incumbent market power. Here, we let the markup parameter (γ) move separately from the vertical step size, acknowledging that markups also may depend on institutions. A rise in the markup (i.e., a decline in γ) will cause the labor share to decline. This follows because, again, we have a balanced growth path that still features a less than unitary elasticity of substitution between capital and labor. A larger markup reduces the size of the capital stock, and this reduces the labor share. Thus, we have a simple link between increased market power and a declining labor share of income. However, from a growth point of view, a rise in the markup increases innovation incentives, and the growth rate should then increase. Thus, the markup story must be counterbalanced by other forces to further match the declining growth rate.[37]

IV. Conclusion

This paper seeks to resolve key tensions between micro and macro descriptions of technological progress. Specifically, the model engages the microeconomic regularities of capital-embodied technological progress and the macroeconomic regularities of balanced growth. By engaging two salient dimensions of innovation—both extensive and intensive advances in capital-embodied technologies—balanced growth emerges in a surprisingly tractable framework. These two frontiers engage in a "tug-of-war," with their balance determining the income shares in a transparent manner while allowing steady growth.

In addition to providing a novel solution to the puzzle of the Uzawa steady-state growth theorem, the model can also inform economic dynamics. The interplay of the two technology frontiers can provide insights about shifts in growth rates and income shares as well as industry dynamics, with applications to structural change (on or off a balanced growth path) and general-purpose technologies. With exogenous technological progress, the model engages such dynamics in a straightforward manner and indeed is flexible enough to fit macroeconomic and sectoral dynamics exactly. The endogenous growth model, which is necessarily more constrained,

[36] The profit share is, examining (44) and (47), linear in $\mu(\phi)/\phi$, so that although the flow profit term $\mu(\phi)$ in the numerator is declining with smaller vertical step sizes, the term ϕ in the denominator is also declining, and this latter effect dominates, so that the profit share of GDP rises. The profit share also rises if new automation technologies have lower productivity (h declines). Thus, one can link innovation getting harder to not only a growth rate decline and labor share decline but also a profit share increase.

[37] Skeptics about the growth slowdown, who may see it as a measurement artifact, may not see a tension here.

delivers further insights about why a balanced growth path would emerge amid the tug-of-war between horizontal and vertical progress. The endogenous growth model may further inform long-run shifts in growth rates and income shares from the perspective of underlying economic parameters.

The results and intuitions of the model build in part from a less than unitary elasticity of substitution between capital and labor. This feature puts the two technology margins in opposition in the evolution of income shares, allowing for balanced growth, while also allowing for intuitive dynamics in growth and income shares, and further applications to structural change, where sectors with relatively slow technological advance take on a larger share of the economy, as with Baumol's cost disease. Further applications engage the macroeconomic implications of general-purpose technologies and the ultimately limited aggregate effects of remarkable technologies, like computing.

Numerous extensions are possible. First, we have not emphasized policy implications, but, for example, the role of tax policy would be nuanced and distinct, with capital taxation having opposing implications for labor shares depending on whether it limits horizontal or vertical progress. Second, to emphasize the novel forces and intuitions at work in this model, we have focused on capital-embodied technological change, but future extensions can incorporate heterogeneous labor, human capital, and capital-skill complementarity to engage additional forms of productivity growth as well as skill-biased technical change. Third, the tractability of the technology dynamics may also allow extensions to business cycles, where frictions may lead to less than full employment and technological change can then have additional labor market effects. Fourth, alternative forms of the endogenous growth theory may prove insightful. Overall, micro-foundations emphasizing the two dimensions of capital-embodied technology advance appear to provide a tractable and rich framework for engaging economic growth and related phenomena.

Appendix A

The proofs and details for Proposition 1, Corollary 1, Cases 1–3, and labor-augmenting progress appear here, following their order in the text. All further proofs and the data analysis details are available in the online Appendix.

PROPOSITION 1 (Balanced Growth Path)*: Under Condition 1, a balanced growth path exists where the capital share is* $s = \beta_t Z_t^{-1}$ *and the growth rate in per capita output, per capita consumption, per capita capital stock, and the wage are* $g = [(\rho - 1)/\rho] q^h$*. The discount rate is* $r = \omega + \theta[(\rho - 1)/\rho] q^h$*.*

PROOF:

On the production side, firm optimization presents explicit solutions at each point t for intermediate outputs and prices ($y_t(i)$ and $p_t(i)$), factor inputs ($l_t(i)$ and $x_t(i)$), aggregate GDP and investment (Y_t and I_t), and wages (w_t). Specifically, Y_t, w_t, and $l_t(i)$ are given explicitly in terms of exogenous variables in Section ID and in (19) and (20). The explicit solution for $x_t(i)$ in terms of exogenous variables follows from (15) and replacing Y_t using (19). Similarly, one finds explicit solutions in terms of endogenous variables for $y_t(i)$, $p_t(i)$, and I_t in a straightforward manner.

On the consumption side, household optimization gives the Euler equation (8). Market clearing further implies that consumption equates to wages in this baseline model. Specifically, household income is $Y_t = c_t L_t + S_t$, and factor payments total $Y_t = w_t L_t + \psi X_t$. Savings equals investment, $S_t = I_t$, and the full depreciation each period equates capital factor payments to gross investment, $I_t = \psi X_t$. Hence, equating income and expenditure, $c_t = w_t$.[38] Per Condition 1, the growth rate in per capita income is $g = \left[(\rho - 1)/\rho\right] q^h$, and with a constant labor share, we have $g_c = g_w = g$. The Euler equation thus implies $r = \omega + \theta g = \omega + \theta\left[(\rho - 1)/\rho\right] q^h$. The transversality condition is $\omega - n > (1 - \theta) g$, which is satisfied by the assumption in Condition 1. ∎

COROLLARY 1 (Limited Innovation, Limitless Growth): *On the balanced growth path, the technology indices shrink at constant rates,* $g_{1-\beta_t} = g_{1/s - Z_t} = -q^h$, *but grow at shrinking rates,* $g_{\beta_t} = g_{Z_t} = \left[(1 - \beta_t)/\beta_t\right] q^h$, *and approach finite limits,* $\beta_t \to 1$ *and* $Z_t \to 1/s$.

PROOF:

From (BGP2), we require $g_{1-\beta_t} = -q^h$. It follows that $g_{\beta_t} = \dot{\beta}_t/\beta_t = \frac{1 - \beta_t}{\beta_t} \frac{\dot{\beta}_t}{1 - \beta_t} = \frac{1 - \beta_t}{\beta_t} q^h$. From (BGP1), we require $\beta_t / Z_t = s$. Therefore, $Z_t \to 1/s$ as $\beta_t \to 1$. Further, $g_{\beta_t} = g_{Z_t}$. The growth rate at which Z_t approaches its limit is $g_{1/s - Z_t} = -\frac{\dot{Z}_t}{1/s - Z_t} = -\frac{\dot{Z}_t}{Z_t} \frac{Z_t}{1/s - Z_t} = -\frac{1 - \beta_t}{\beta_t} q^h \frac{1}{1/\beta_t - 1} = -q^h$. ∎

Case 1 (A Micro-innovation Process for Balanced Growth)**:** Let existing tasks increase their productivity proportionally by an amount ϕ in expectation, with hazard rate q^v. Then a BGP will occur so long as newly automated tasks have productivity $z_t(\beta_t) = h(1 - \beta_t)$, where $h \geq 1/(1 - s)$ and $s = q^h/(q^v \phi h)$.

PROOF:

Differentiate the first BGP condition in the form (26) using Leibniz's rule. This gives

$$\int_0^{\beta_t} \frac{\dot{z}_t(i)}{z_t(i)^2} di = \frac{\dot{\beta}_t}{z_t(\beta_t)}. \tag{48}$$

In the vertical innovation process, an increase in a line's productivity occurs with hazard rate q^v, bringing a new productivity level $\left[1 + \phi_t(i)\right] z_t(i)$. In expectation, we have assumed (see text) that $E_t\left[\phi_t(i)\right] = \phi$. Therefore, $E_t\left[\dot{z}_t(i)\right] = q_v \phi z_t(i)$, and

$$E_t\left[\int_0^{\beta_t} \frac{\dot{z}_t(i)}{z_t(i)^2} di\right] = \phi q^v \int_0^{\beta_t} \frac{1}{z_t(i)} di = s \phi q^v.$$

[38] This will not be true in the endogenous technology model of Section III, for the usual reason in endogenous growth where capital inputs still fully depreciate but there are profits from market power adding to household income and, on the expenditure side, there are additional investments in R&D that drive technological progress.

Meanwhile, the horizontal process of automated new tasks occurs as the rate $\dot{\beta}_t = q^h(1-\beta_t)$ on the BGP. Thus, we can write the BGP condition (A1) as

$$\frac{q^h(1-\beta_t)}{z_t(\beta_t)} = s\phi q^v.$$

Rearranging this produces the necessary condition of Corollary 1, equation (26), governing the productivity draws for new technologies. Namely, with $z_t(\beta_t) = h(1-\beta_t)$, we have $s = q^h/(q^v\phi h)$.

Next, consider the condition that all automated technologies be adopted (as opposed to using labor). First note that the automated technology with the lowest productivity will be the newly automated one. This follows because, from (26), the initial productivity is declining on the growth path and, once automated, the productivity on any line can only be increasing. Thus, all automated technologies will be used so long as the newly automated technology is used. Second, from (21), the newly automated technology will be adopted if $z_t(\beta_t) \geq (1-\beta_t)/(1-s)$. Using (26), this condition is $h \geq 1/(1-s)$, with $s = q^h/(q^v\phi h)$ as above. ∎

Case 2 (A Micro-innovation Process with Stochastic Step Sizes)**:** Let the hazard rate for successful vertical innovation be q^v with new productivity drawn from a Pareto distribution with shape parameter $\alpha^v > 1$. Similarly, let the hazard rate for successful horizontal innovation be q^h with initial productivity drawn from a Pareto distribution with shape parameter $\alpha^h > 1$. Then there is a balanced growth path.

PROOF:

A vertical technological advance occurs with hazard rate q^v. The new productivity level, conditional on success (i.e., beating the existing automated technology level on that line), is drawn from a Pareto distribution, $f(z) = \alpha^v\left[z_t(i)\right]^{\alpha^v}/z^{\alpha^v+1}$. The expected new productivity level on a given line, conditional on success, is then $\left[\alpha^v/(\alpha^v-1)\right]z_t(i)$. Proportionally, the expected step size is then

$$E_t\left[\phi_t(i)\right] = \frac{1}{\alpha^v - 1}. \tag{49}$$

A horizontal technological advance occurs with hazard rate q^h. The initial productivity draw, conditional on success (i.e., beating labor), is given by a Pareto distribution, $f(z) = \alpha^h[z_t^{min}]^{\alpha^h}/z^{\alpha^h+1}$. The expected initial productivity level is then

$$E_t\left[z_t(\beta_t)\right] = h(1-\beta_t), \tag{50}$$

where $h = \left[\alpha^h/(\alpha^h-1)\right]\left[1/(1-s)\right]$.

We see that we satisfy the conditions for a BGP, according to the same reasoning as Case 1. Namely, taking expectations in (48), we have constants on both sides and a solution for the capital share, s, in terms of the innovation parameters $\{q^v, q^h, \alpha^v, \alpha^h\}$. Thus, we satisfy (BGP1). With horizontal innovation at rate q^h, we satisfy (BGP2). ∎

Case 3 (Heterogeneous Technological Advance)**:** Let there be different subintervals of tasks indexed $j \in \{1, \ldots, J\}$. Let each subinterval have a measure of tasks u^j and an automation share $\beta_t^j < u^j$, where the u^j sum to 1 and the β_t^j sum to β_t. Let vertical technological advance occur at different rates for different subintervals, with hazard rates q^{vj} and expected proportional step sizes ϕ^j in a given subinterval. Let automation of the nonautomated tasks in each subinterval occur at rate q^h, and let the initial productivity of a newly automated technology in a given subinterval be $h^j\left(u^j - \beta_t^j\right)$, where h^j is a sufficiently large constant. Then there is a balanced growth path.

PROOF:

First consider (BGP2). We assume that the automation of the nonautomated tasks in each subinterval occurs at rate q^h. That is, $\dot{\beta}_t^j = q^h\left(u^j - \beta_t^j\right)$. Summing across the subintervals, we have $\dot{\beta}_t = q^h(1 - \beta_t)$. Thus, we satisfy (BGP2).

Next consider (BGP1) in its task-summation form, (26). A sufficient condition for (26) to hold is

$$\int_0^{\beta_t^j} z_t(i)^{-1}\, di = s^j \tag{51}$$

for each subinterval.[39] Following the procedure in Case 1 above, differentiate this condition with respect to time. Use the assumption that vertical technological advance occurs with hazard rate q^{vj} and expected proportional step size ϕ^j for a given subinterval. Then we find

$$\frac{q^h\left(u^j - \beta_t^j\right)}{z_t\left(\beta_t^j\right)} = s^j \phi^j q^{jv}.$$

Further use the assumption that the initial productivity of newly automated technology in a given subinterval follows $z_t\left(\beta_t^j\right) = h^j\left(u^j - \beta_t^j\right)$. We see directly that we satisfy (51), with $s^j = q^h/\left(q^{vj}\phi^j h^j\right)$, a constant, for each subinterval. Thus, we satisfy (BGP1) in its task summation form.

Finally, consider the technology adoption condition. We require the h^j to be large enough so that newly automated technologies will be adopted. This requirement is $z_t\left(\beta_t^j\right) \geq z_t^{\min}$, or $h^j\left(u^j - \beta_t^j\right) \geq (1 - \beta_t)/(1 - s)$, which must hold for all subintervals. Note that with $g_{u^j - \beta_t^j} = g_{1-\beta_t} = -q^h$, the ratio $\left(u^j - \beta_t^j\right)/(1 - \beta_t)$ is a constant along the growth path. Thus, the technology adoption condition on the BGP is

$$h^j \geq \frac{c^j}{1 - s} \tag{52}$$

for all j, where $c^j = \left(u^j - \beta_0^j\right)/(1 - \beta_0)$. ∎

[39] The unit measure of tasks in the overall economy is now J submeasures, which are separately indexed, each with its own measure u^j and each with its own automated task share β_t^j.

Labor-Augmenting Technological Progress.—Here, we consider an extension where labor productivity changes over time. In particular, we allow for a time-varying A_t instead of the constant A. The intermediates production function (5) becomes

$$y_t(i) \;=\; \begin{cases} A_t l_t(i), & \text{for all } i \;\in\; [0,1]; \\ z_t(i)^{\frac{\rho-1}{\rho}} x_t(i), & \text{for all } i \;\in\; [0,\beta_t]; \end{cases}$$

and the technology adoption condition (14) becomes

$$\frac{w_t}{A_t} \;\geq\; \psi z_t(i)^{\frac{1-\rho}{\rho}}.$$

Following the same development as in the main text, assume this adoption condition holds for all automated technologies. Then we find expressions for GDP

$$Y_t \;=\; \upsilon A_t\left(1 \;-\; \beta_t Z_t^{-1}\right)^{-1/\rho}\left(1 \;-\; \beta_t\right)^{\frac{1-\rho}{\rho}} L_t$$

as in (19) in the main text and for the equilibrium wage,

$$w_t \;=\; \upsilon A_t\left(1 \;-\; \beta_t Z_t^{-1}\right)^{\frac{\rho-1}{\rho}}\left(1 \;-\; \beta_t\right)^{\frac{1-\rho}{\rho}}$$

as in (20), but now with the distinction that we have the time-varying A_t instead of the constant A. The capital share is again $s_{K_t} \;=\; \beta_t Z_t^{-1}$, and other results for prices and quantities follow as in the main text.

Now confirm the technology adoption condition. We see immediately that the ratio w_t/A_t between the wage and labor productivity is the same as before, and hence, the technology adoption condition follows as before.

We see directly in the GDP or wage expressions above that the steady-state growth rate becomes $g \;=\; \left[(\rho \;-\; 1)/\rho\right] q^h \;+\; g_A$.

Altogether, the labor-augmenting technology path A_t does not affect the technology adoption condition or the labor share. It affects wages and aggregate output in a linear way, and per capita income growth now also depends on growth in A_t.

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